Sensor Networks Tricks

Sensor Fusion

Fredrik Gustafsson
fredrik.gustafsson@liu.se

Gustaf Hendeby
gustaf.hendeby@liu.se

Linköping University

Sensor Networks

Summary of nonlinear models of the form $y_k = h(x) + e_k$ in sensor networks

$$\begin{aligned}
\text{TOA} \quad & r_k = \|x - p_k\| + e_k \\
\text{TDOA} \quad & r_k = \|x - p_k\| + r_0 + e_k \\
\text{DOA} \quad & \varphi_k = \operatorname{arctan2}(x_2 - p_{k,2}, x_1 - p_{k,1}) + e_k \\
\text{RSS} \quad & y_k = P_0 - \beta \log(\|x - p_k\|)
\end{aligned}$$

Nonlinear estimation theory from Chapter 3 gives a whole toolbox of solutions.

However, for this particular application, there is a number of dedicated solutions and tricks:

- Interpreting TDOA as hyperbolic functions.
- Special tricks for TOA
- Triangulation of DOA
- RSS simultaneous localization and propagation parameter estimation

TDOA Geometry

TOA corresponds to circles that intersect at the transmitter, but what is TDOA?
Common offset r_0 (due to unsynchronized clocks)

$$r_k = \|x - p_k\| + r_0, \quad k = 1, 2, \ldots, N.$$

Estimation approach:
Consider r_0 as a parameter (*cf.* GPS).

Common in literature:
Study range differences

$$r_{i,j} = r_i - r_j, \quad 1 \leq i < j \leq N.$$

Gives nice geometric interpretation.

TDOA: maths

Assume $p_1 = (D/2, 0)^T$ and $p_2 = (-D/2, 0)^T$, respectively, then

$$\begin{aligned} r_1 &= \sqrt{x_2^2 + (x_1 - D/2)^2} \\ r_2 &= \sqrt{x_2^2 + (x_1 + D/2)^2} \\ r_{12} &= r_2 - r_1 = h(x, D) \\ &= \sqrt{x_2^2 + (x_1 + D/2)^2} - \sqrt{x_2^2 + (x_1 - D/2)^2}. \end{aligned}$$

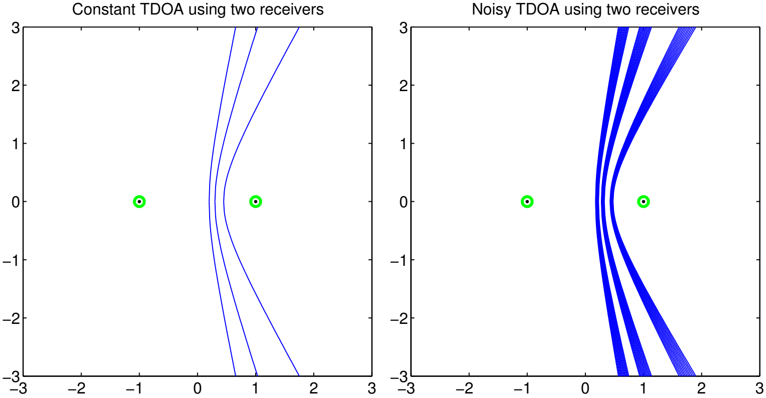
Simplify

$$\frac{x_1^2}{a} - \frac{x_2^2}{b} = \frac{x_1^2}{r_{12}^2/4} - \frac{x_2^2}{D^2/4 - r_{12}^2/4} = 1.$$

This is the definition of a hyperbolic function!

TDOA: illustration

- Illustration of three different values of r_{12}.
- Corresponds to three different hyperbolic functions.
- Measurement noise on r_{12} gives confidence bands that becomes thicker with distance.



Direction of Arrival (DOA)

The solution to this hyperbolic equation has asymptotes along the lines

$$x_2 = \pm \frac{b}{a} x_1 = \pm \sqrt{\frac{D^2/4 - r_{12}^2/4}{r_{12}^2/4}} x_1 = \pm x_1 \sqrt{\left(\frac{D}{r_{12}}\right)^2 - 1}.$$

AOA, φ, for far-away transmitters

$$\varphi = \pm \arctan\left(\sqrt{\left(\frac{D}{r_{12}}\right)^2 - 1}\right).$$

Thus, if the transmitter is far away from the network, each TDOA pair can be seen as a DOA and triangulation can be applied.

DOA Triangulation

Angle observations from sensor at position p_k

$$\varphi_k = \arctan\left(\frac{x_2 - p_{k,2}}{x_1 - p_{k,1}}\right)$$

$$(x_1 - p_{k,1})\tan(\varphi_k) = x_2 - p_{k,2}.$$

Linear model follows immediately,

$$\mathbf{y} = \mathbf{H}x + \mathbf{e}$$

$$\mathbf{y} = \begin{pmatrix} p_{1,1}\tan(\varphi_1) - p_{1,2} \\ p_{2,1}\tan(\varphi_2) - p_{2,2} \\ \vdots \\ p_{N,1}\tan(\varphi_N) - p_{N,2} \end{pmatrix}, \qquad \mathbf{H} = \begin{pmatrix} \tan(\varphi_1) & -1 \\ \tan(\varphi_2) & -1 \\ \vdots \\ \tan(\varphi_N) & -1 \end{pmatrix}.$$

Note:

What is the relation between the measurement noise on φ_k and $\mathbf{e}$ here?

Dedicated Explicit LS Solutions

Basic trick: study NLS of *squared* distance measurements:

$$\hat{x} = \arg\min_x \sum_{k=1}^{N} \left(r_k^2 - \|x - p_k\|^2\right)^2.$$

If the terms to be squared are expanded, it looks like quadratic terms $\|x\|^2$ would appear. However, there are a couple of tricks to get rid of these to get a quadratic cost function whose minimum has an analytical solution.

- TOA range parameter trilateration: r_k^2 is linear in $(x_1, x_2, x_1^2 + x_2^2)^T$
- TOA reference sensor trilateration: $r_k^2 - r_1^2$ is linear in x

Note:

What is the relation between the measurement noise on r_k and the implicitly assumed additive noise on r_k^2?

RSS (1/2)

Received signal strength (RSS) observations:

- All waves (radio, radar, IR, seismic, acoustic, magnetic) decay exponentially in range.
- Receiver k measures energy/power/signal strength for wave i:

$$P_{k,i} = P_{0,i}\|x - p_k\|^{\beta_i}.$$

- Transmitted signal strength and path loss constant may be unknown.
- Communication constraints make coherent detection (that is, TOA and TDOA measurements) from the signal waveform impossible.
- Solution: Compare $P_{k,i}$ for different receivers.

RSS (2/2)

Log model:

$$\bar{P}_{k,i} = \bar{P}_{0,i} + \beta_i \underbrace{\log(\|x - p_k\|)}_{=:c_k(x)}$$
$$y_{k,i} = \bar{P}_{k,i} + e_{k,i}$$

Use separable least squares to eliminate path loss constant and transmitted power for wave i:

$$\widehat{(x,\theta)} = \arg\min_{x,\theta} V(x,\theta)$$
$$V(x,\theta) = \sum_{i=1}^{M} \sum_{k=1}^{N} \frac{\big(y_{k,i} - h(c_k(x),\theta_i)\big)^2}{\sigma_{P,i}^2}$$
$$h(c_k(x),\theta_i) = \theta_{i,1} + \theta_{i,2} c_k(x)$$
$$c_k(x) = \log(\|x - p_k\|)$$

Finally, use NLS to optimize over 2D target position x.

Summary

- The basic network measurements:

$$
\begin{aligned}
\text{TOA} \quad & r_k = \|x - p_k\| + e_k \\
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\text{DOA} \quad & \varphi_k = \text{arctan2}(x_2 - p_{k,2}, x_1 - p_{k,1}) + e_k \\
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\end{aligned}
$$

  NLS or NLT general approaches to estimate x.

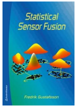
- Tricks (not statistically optimal!) described in this lecture:

  TDOA pairwise differences correspond to hyperbolic functions

  TOA range parameter trilateration: r_k^2 is linear in $(x_1, x_2, x_1^2 + x_2^2)^T$

  TOA reference sensor trilateration: $r_k^2 - r_1^2$ is linear in x

  DOA triangulation approach: x is an affine function in $\tan(\varphi_k)$



Sections 4.3–4.6